**CURRICULUM VITAE DI MARIA MANFREDINI**

## Academic curriculum

Current position: since September 2019 Associate Professor of Mathematical Analysis at the University of Modena and Reggio Emilia.

- September 2014 Associate Professor of Mathematical Analysis at the University of Bologna.

- July 1995: Researcher in the Mat/05 group at the University of Bologna.

- June 1995: PhD in Mathematics

- June 1990: Graduate in Mathematics from University of Modena with the vote of 100/100 cum laude

## Institutional activities

- from February 2020, Head of the FIM Department for Equal Opportunities;
- from April 2022 to the Paritetica Commission of the FIM Department;
- from April 2022 contact person of the FIM Department for orientation towards the University.

## Main research interests

My research activity was mainly carried out in the study Second order partial differential differentials Degenerate, linear and non-linear. More precisely it concerned the study of equations that can be written as the sum of squares of vector fields, Regularity problems of solutions and problems of geometric theory of measurement in Riemannian and sub-Riemannian structures. There is a rich literature for operators defined by regular fields, but there is no general theory for nonlinear or nonregular fields. A large part of my recent research work has been devoted to to develop a theory for such operators.

## List of Publications

**Asymptotic estimates for variational equations.**

[1] M. Manfredini, *Asymptotic behavior of solutions of variational equations*, Rend. Circ. Mat. Palermo, II. Ser. 41, No.3, (1992) 441-465.

[2] G. Leoni, M. Manfredini, P. Pucci, *Stability properties for solutions of general Euler-Lagrange systems*, Differ. Integral Equ. 5, No.3, (1993) 537-552.

**Kolmogorov type equation**

[3] M. Bramanti, M. C. Cerutti, M. Manfredini, *$L^p$ estimates for some ultraparabolic operators with discontinuous coefficients*, J. Math. Anal. Appl. 200, No.2, (1996) 332-354.

[4] M. Manfredini, *The Dirichlet problem for a class of ultraparabolic equation*, Adv. Diff. eq., 2, (1997) 831-866.

[5] M. Manfredini, S. Polidoro, *Interior regularity for weak solutions of ultraparabolic equations in divergence form with discontinuous coefficients*, Boll. Unione Mat. Ital., Sez. B, Artic. Ric. Mat. (8) 1, No.3, (1998) 651-675.

**Measure theory in Lie group.**

[6] G. Citti, M. Manfredini, *Blow-Up in Non-Homogeneous Lie Groups and Rectifiability*, Houston Journal of Mathematics, 31, No. 2, (2005) 333-353.

[7] G. Citti, M. Manfredini, *Implicit function theorem in Carnot-Carathéodory spaces*, Commun. Contemp. Math. 8 no. 5, (2006) 657-680.

**A priori estimates for degenerate equations.**

[8] M. Manfredini, *Compact embedding theorems for generalized Sobolev spaces*, Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Nat., IX. Ser., Rend. Lincei, Mat. Appl. 4, No.4, (1993) 251-263.

[9] M. Manfredini, A. Pascucci, *A priori estimates for quasilinear degenerate parabolic equations*, Proceedings of the American Mathematical Society, 131 (2003) 1115-1120

[10] G. Citti, M. Manfredini, *Uniform estimates of the fundamental solution for a family of hypoelliptic operators*, Potential Anal., 25, (2)(2006) 147-164.

[11] M. Manfredini, *Uniform Schauder estimates for regularized hypoelliptic equations*, Ann. Mat. Pura ed Applicata, 188, (2009) 417-428.

**Riemannian motion for curvature.**

[12] G. Citti, M. Manfredini, *Long time behavior of Riemannian mean curvature flow*, Journal of Mathematical Analysis and Applications, 273, (2002) 353-369.

[13] G. Citti, M. Manfredini, *A degenerate parabolic equation arising in image processing*, Communication on Applied Analysis, 8, 1, (2004) 125-141.

**The Mumford-Shah functional.**

[14] A. Sarti, G. Citti, M. Manfredini, *From neural oscillations to variational problems in the visual cortex*, Journal of Physiology Paris, vol. 97, n.2-3, (2003) 379-385.

[15] G. Citti, M. Manfredini, A. Sarti, *Neuronal oscillation in the visual cortex: Gamma-convergence to the Riemannian Mumford-Shah functional*, SIAM Journal Mathematical Analysis, vol. 36, n.6, (2004) 1394-1419.

[16] G. Citti, M. Manfredini, A. Sarti, *Finite difference approximation of the Mumford and Shah functional in a contact manifold of the Heisenberg space*, Commun. Pure Appl. Anal. 9, no. 4,(2010) 905-927.

**Mean curvature equation in Heisenberg group.**

[17] L. Capogna, G. Citti, M. Manfredini, *Regularity of minimal surfaces in the one-dimensional Heisenberg group*, Bruno Pini Mathematical Analysis Seminar, University of Bologna Department of Mathematics: Academic Year 2006/2007 (Italian), 147-162, Tecnoprint, Bologna, 2008.

[18] L. Capogna, G. Citti, M. Manfredini, *Smoothness of Lipschitz minimal intrinsic graphs in Heisenberg groups* $\mathbb{H}^n$, $n > 1$, J. Reine Angew. Math. 648, (2010) 75-110.

[19] L. Capogna, G. Citti, M. Manfredini, *Regularity of non-characteristic minimal graphs in the Heisenberg group* $\mathbb{H}^1$, Indiana Univ. Math. J. 58, no. 5, (2009) 2115-2160.

[20] L. Capogna, G. Citti, M. Manfredini, *Uniform Gaussian bounds for subelliptic heat kernels and an application to the total variation flow of graphs over Carnot groups*, Anal. Geom. Metr. Spaces 1 (2013), 255-275.

**Vector fields with rough coefficients.**

[21] M. Manfredini, *Step-two nonsmooth vector fields: the Poincaré inequality and the fundamental solution of the associated operator*, Bruno Pini Mathematical Analysis Seminar, University of Bologna Department of Mathematics: Academic Year 2008/2009 (Italian), 111-123, Tecnoprint, Bologna, 2009.

[22] M. Manfredini, *A note on the Poincaré inequality for Lipschitz vector fields of step two*, Proc. Amer. Math. Soc. 138, no. 2, (2010) 567-575.

[23] M. Manfredini, *Fundamental solutions for sum of square vector fields operators with $C^{1,\alpha}$ coefficients*, Forum Mathematicum 24, Issue 5, (2012) 973-1011.

[24] G. Citti, M. Manfredini, A. Pinamonti, F. Serra Cassano, *Smooth approximation for intrinsic Lipschitz functions in the Heisenberg group*, Calc. Var. Partial Differential Equations 49 (2014), no. 3-4, 1279-1308.

[25] L. Capogna, G. Citti, M. Manfredini, Uniform Gaussian Bounds for Subelliptic Heat Kernels and an Application to the Total Variation Flow of Graphs over Carnot Groups. ANALYSIS AND GEOMETRY IN METRIC SPACES, vol. 1, (2013), p. 255 -275.

[26] L. Capogna, G. Citti, M. Manfredini, Regularity of mean curvature flow of graphs on Lie groups free up to step 2. NONLINEAR ANALYSIS, vol. 126, (2015), p. 437-450.

[27] Bonfiglioli A., Citti G., Cupini G., Manfredini M., Montanari A., Morbidelli D., Pascucci A., Uguzzoni F., Polidoro S. The Role of Fundamental Solution in Potential and Regularity Theory for Subelliptic PDE. In: Geometric Methods in PDE. SPRINGER INDAM SERIES, vol. 13, p. 341-373, Springer, (2015).

[28] G. Citti, M. Manfredini, A. Pinamonti, F. Serra Cassano, Poincaré-type inequality for Lipschitz continuous vector fields in the Heisenberg group, JOURNAL DE MATHMATIQUES PURES ET APPLIQUES, vol. 103, (2016), p. 265-292,

[29] M. Bramanti, L. Brandolini, M. Manfredini, M. Pedroni, Fundamental solutions and local solvability for nonsmooth Hormander's operators,Mem. Amer. Math. Soc. 249 (2017), no. 1182, v + 79 pp.

**Recent articles.**

[30] M. Manfredini, *Intrinsic fractional Taylor formula* , Bruno Pini Mathematical Analysis Seminar, 12(1), (2021). 1–14.

[31] M. Manfredini, M. Piccinini, S. Polidoro , *The Dirichlet problem for a family of totally degenerate differential operators*, subject to review.

[32] M. Manfredini, M. Piccinini, S. Polidoro, G. Palatucci, *Hölder continuity and boundedness estimates for nonlinear fractional equations in the Heisenberg group*, accepted for publication.

[33] G. Citti, M. Manfredini, Y. Sire *Hölder regularity for weak solutions of Hörmander type operators*, preprint.

## Recent teaching activity

**A.A. 2011/2012**: Titolare del Corso di Analisi Matematica L-B del corso di Laurea di Ingegneria Gestionale (L-Z) (6 crediti formativi, 60 ore di lezione).

**A.A. 2012/2013**: - Titolare del Corso di Analisi 1 del corso di Laurea di Ingegneria Edile-Architettura (6 crediti formativi, 80 ore di lezione).

- Modulo di 30 ore su 60 per il Corso di Analisi Matematica L-B del corso di Laurea di Ingegneria Gestionale (L-Z) (30 ore di lezione).

**A.A. 2013/2014**: - Titolare del Corso di Analisi 2 del corso di Laurea di Ingegneria Edile-Architettura (6 crediti formativi, 60 ore di lezione).

- Modulo di 45 ore su 90 per il Corso di Analisi Matematica T-2 del corso di Laurea di Ingegneria Edile (sede di Ravenna) (45 ore di lezione).

**A.A. 2014/2015**:

- Titolare del Corso di Analisi Matematica T-1 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (9 crediti formativi, 90 ore di lezione).

- Titolare di un Modulo di 45 ore su 90 per il Corso di Analisi Matematica T-2 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (45 ore di lezione).

**A.A. 2015/2016**:

- Titolare del Corso di Analisi Matematica T-1 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (9 crediti formativi, 90 ore di lezione).

- Modulo di 30 ore su 90 per il Corso di Analisi Matematica T-2 del corso di Laurea di Ingegneria Laurea in Ingegneria dell'automazione e di Laurea in Ingegneria dell'energia elettrica

**A.A. 2016/2017**:

- Titolare del Corso di Analisi Matematica T-1 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (9 crediti formativi, 90 ore di lezione).

- Titolare del Corso di Analisi Matematica T-B del corso di Laurea di Ingegneria Gestionale (6 crediti formativi, 60 ore di lezione).

**A.A. 2017/2018**:

- Titolare del Corso di Analisi Matematica T-1 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (9 crediti formativi, 90 ore di lezione).

- Titolare del Corso di Analisi Matematica T-B del corso di Laurea di Ingegneria Gestionale (6 crediti formativi, 60 ore di lezione).

- Titolare dell'Insegnamento "'Partial Differential Equations" per il Corso del Dottorato di Matematica di Bologna, XXXIII ciclo, (30 ore di lezione frontali).

**A.A. 2018/2019**:

- Titolare del Corso di Analisi Matematica T-2 del corso di Laurea di Ingegneria Edile (Campus di Ravenna) (9 crediti formativi, 90 ore di lezione).

- Titolare del Corso di Analisi Matematica T-B del corso di Laurea di Ingegneria Gestionale (6 crediti formativi, 60 ore di lezione).

- Ciclo di lezioni "Partial Differential Equations" per il Corso del Dottorato di Matematica di Unimore (18 ore)

- Titolare del Corso di Analisi Matematica I del corso di Laurea di Ingegneria Informatica Unimore (9 crediti formativi, 81 ore di lezione).

**A.A. 2019/2020**:

- Titolare del Corso di Analisi Matematica I del corso di Laurea di Ingegneria Informatica (9 crediti formativi, 81 ore di lezione).

- Titolare del corso di Complementi di analisi matematica di Fisica e del corso di Analisi matematica B di Matematica (6 crediti formativi, 48 ore di lezione).

- Modulo di 54 ore su 81 del corso di Analisi Matematica II per Ingegneria del Veicolo

- Ciclo di lezioni per il corso "Hypoelliptic Partial Differential Equations" per il Corso del Dottorato di Matematica (18 ore)

**A.A. 2020/2021**:

- Titolare del Corso di Analisi Matematica I del corso di Laurea di Ingegneria Informatica (9 crediti formativi, 81 ore di lezione).

- Titolare del corso di Complementi di analisi matematica di Fisica e del corso di Analisi matematica B di Matematica, laurea triennale (6 crediti formativi, 48 ore di lezione).

- Modulo di 40 ore su 90 per corso di Analisi Matematica II Accademia Militare di Modena

-Modulo di 18 ore su 36 per il corso di Equazioni alle derivate parziali, laurea Magistrale di Matematica

- Ciclo di lezioni per il corso "Hypoelliptic Partial Differential Equations" per il Corso del Dottorato di Matematica (18 ore)

**A.A. 2021/2022**:

- Titolare del Corso di Analisi Matematica I del corso di Laurea di Ingegneria Informatica (9 crediti formativi, 81 ore di lezione).

- Titolare del corso di Complementi di analisi matematica di Fisica e del corso di Analisi matematica B di Matematica, laurea triennale (6 crediti formativi, 48 ore di lezione).

- Modulo di 27 ore su 81 del corso di Analisi Matematica II per Ingegneria del Veicolo

- Modulo di 50 ore su 90 per corso di Matematica II Accademia Militare di Modena

- Ciclo di lezioni per il corso "Hypoelliptic Partial Differential Equations" per il Corso del Dottorato di Matematica (18 ore)

**A.A. 2022/2023**:

- Titolare del Corso di Analisi Matematica I del corso di Laurea di Ingegneria Informatica (9 crediti formativi, 81 ore di lezione).

- Titolare del corso di Complementi di analisi matematica di Fisica e del corso di Analisi matematica B di Matematica, laurea triennale (6 crediti formativi, 48 ore di lezione).

- Modulo di 27 ore su 81 del corso di Analisi Matematica II per Ingegneria del Veicolo

- Modulo di 50 ore su 90 per corso di Matematica II Accademia Militare di Modena

- Ciclo di lezioni per il corso "Hypoelliptic Partial Differential Equations" per il Corso del Dottorato di Matematica (18 ore)

- Modulo di 18 ore su 36 per il corso di Equazioni alle derivate parziali, laurea Magistrale di Matematica

**A.A. 2023/2024**:

- Titolare del Corso di Analisi Matematica II del corso di Laurea di Ingegneria del Veicolo (9 crediti formativi, 81 ore di lezione).

- Titolare del corso di Complementi di analisi matematica di Fisica e del corso di Analisi matematica B di Matematica, laurea triennale (6 crediti formativi, 48 ore di lezione).

- Modulo di 45 ore su 90 per corso di Matematica I Accademia Militare di Modena

- Modulo di 50 ore su 90 per corso di Matematica II Accademia Militare di Modena

- Ciclo di lezioni per il corso "Hypoelliptic Partial Differential Equations" per il Corso del Dottorato di Matematica (18 ore)

## Brief description of the most recent publications

The main results I have obtained concern nonlinear differential equations of type Hörmander, and in particular equations of mean curvature. I then studied:

(i) properties of regular or rectifiable surfaces in groups;

(ii) ownership of basic solutions of type Hörmander designed as limits of elliptical operators or operators with non smooth coefficients;

(iii) Asymptotic properties of mean curvature motion solutions;

(iv) the regularity of the minimal surfaces and the limit of the minimum riemannian surfaces;

(v) Mumford-Shah functional minima in this setting.

These results intervene in primary visual cortex models. In fact, the visual cortex was modeled by Petitot and Tondut in the SE(2) group of the rigid motions of the plane. In this setting, minimal surfaces model subjective surfaces, while differential equations describe the neural propagation of the visual signal.

(i) *Surfaces in non-homogeneous Lie groups.*

Model structures to develop the theory of Hörmander operators in $\mathbb{R}^n$

$$L = \sum_{i=1}^{m} X_i^2 \quad 1 \le m \le n \tag{1}$$

are the Lie groups, in particular, the theory is simplified in the case of homogeneous groups, called Carnot groups, for which natural dilations are defined compatible with group law. The notion of intrinsic regular surface was introduced in the Carnot groups only in 2001 by Franchi, Serapioni and Serra Cassano. A regular surface is the set of zeros of a function whose intrinsic gradient is not null. There is a wide literature, always of the same authors, about the study of surfaces in homogeneous groups.

In [6] an rectifiable result is proved for surfaces in non-homogeneous setting, introducing a notion of frontier reduced similar to that of De Giorgi for the Euclidean case, and Franchi, Serapioni, Serra Cassano for homogeneous groups. The main difficulty is that in non-homogeneous groups are not defined natural dilations, and therefore it is necessary to introduce expansion in advance approximate, which allow to make a blow- up, and then introduce the notions of planes tangents and intrinsical rectifiability. The main result of the work ensures that the reduced (intrinsic) frontier of a Cacciopoli set in the sense of group is rectifiable.

In [7], for surfaces of class $C^1$ intrinsically, an implicit function theorem has been proved. Our result is contemporary, but more general Ambrosio, Serra Cassano and Vittone in the group of Heisenberg. The implicit function $u$ is differentiated from a family of appropriate nonlinear fields

$$X_{i,u} = \sum_{j=1}^{n} a_{ij}(u)\partial_{x_j}. \tag{2}$$

(ii) *A priori estimates and construction of fundamental solutions.*

It is well known that linear operators of type ( 1) have a fundamental solution $\Gamma$, see Rothschild and Stein's 1977 article. On the other hand degenerate operators of the type (1) can be formally thought as limit for $\epsilon \to 0$ of regularization operators Riemannian parameter dependent $\epsilon$:

$$L_\epsilon = L + \epsilon\,\Delta,$$

where $\Delta$ is the laplace operator $\mathbb{R}^n$. In [10] we prove a priori estimates uniform in $\epsilon$ of fundamental solution $\Gamma_\epsilon$ of $L_\epsilon$, and we prove that $\Gamma$ is the limit of $\Gamma_\epsilon$ as $\epsilon \to 0$. The proof of these estimates is based on a different lifting process from the classic one of Rothschild and Stein and consists in lifting $L_\epsilon$ to an independent operator from $\epsilon$, whose fundamental solution provides the estimates searched for the fundamental solutions of $L_\epsilon$. Using this technique uniform a priori estimates were obtained in [11] Schauder type for solution of the problem $L_\epsilon u = f$. In this context, estimates of solutions of a large class of quasilinear parabolic equations in the paper [9].

The operators studied in this work are expressed in terms of fields $C^\infty$. However, in the study of quasilinear equations, fields depend on the solution, and it is not generally possible to suppose that they are regular and not even to think of them simply as perturbation of known linear operators. An axiomatic theory for nonregular fields ff this type is presented by Sawyer and Wheeden, but you cannot apply to fields of type ( 2).

In the work [23] non-linear fields of the type ( 2) with $u$ lipschitz in an intrinsic sense, a local fundamental solution is constructed using the method of the Levi parametric.

In [29] I considered Hörmander operators sum of squares of vector fields non smooth and of step $r$. In this article a fundamental solution of local type is constructed and estimates of its derivatives are provided up to order two. This makes it possible to demonstrate a priori estimates of the Schauder type and to deal with the study of the local risolubility.

Poincaré inequality is known and is an essential tool in the method Moser iterative testing the hölderian regularity of solutions. For regular camps it was tested by Jerison. For non smooth and diagonal vector fields has been demonstrated by Franchi and Lanconelli. More recently, Lanconelli and Morbidelli prove poincaré and under a condition of the spheres of the induced metric. By developing such a method Montanari and Morbidelli prove Poincaré inequality and assuming the condition that the fields and their two order switches are Leipzig-like and generate all the space. However, in many situations this kind of regularity is not satisfied. For example it is not satisfied for vector fields ( 2) They are involved in the study of minimum surfaces in Heisenberg which have only Lipschtz continuous coefficients or for the fields considered by Rios, Sawyer and Wheeden.

The purpose of work [22] is to fill this gap by proving a Poincaré inequality and for fields with only Lipschitz coefficients.

Also in this context, in the work [24] it is shown that the Leipzig functions (intrinsically) can be approximated, together with their gradients, by regular functions. The test consists in providing a regular approximation of the finite perimeter of a set whose boundary is the intrinsic graph of the function to be approximated.

(iii) *Asimptotic estimates for the mean curvature riemannian motion.*

The motion of a surface where each point moves in the normal direction with speed curvature is called motion by curvature. In Euclidean and Riemannian contexts the problem has been extensively studied, mainly by Ecker and Huisken. It is well known that motion solutions for Riemannian curvature with given to the boundary data fixed tend to the solution of the equation of minimum surfaces with the same data. Especially if the baoundary data is zero, the solution tends to zero. In the work [12] we prove that the solution, properly normalized, tends instead to the first carrier of an operator Laplace-Beltrami type, that is, type ( 1) with $n = m$. The result has an interest in image processing and stability applications. Another differential equation describing a geometric flow of fronts, and presenting itself in image processing, is studied in [13]. The main difficulty is due to the fact that the equation is non-local, and proof of the existence and stability of solutions is obtained by a viscosity technique.

(iv) *Mean curvature equations in the Heisenberg group: non linear vector fields.*

The prescribed mean curvature equation in the Heisenberg group $\mathbb{H}^n$ occurs formally as the analogous Riemannian equation, in terms of nonlinear fields of the type (2)

$$\sum_{i=1}^{2n-1} X_{i,u}\left(\frac{X_{i,u}u}{\sqrt{1+|\nabla_{X,u}u|^2}}\right) = f, \quad x \in \Omega \subset \mathbb{R}^{2n} \tag{3}$$

where $\nabla_{X,u} = (X_{1,u}, \ldots, X_{2n-1,u})$. A regular surface is called minimal if its satifies the equation with $f \equiv 0$.

For this problem thought as critical points of the functional perimeter, Garofalo and Nhieu prove the regularity in spaces with bounded variation and Pauls proves the regularity in the Sobolev spaces . Next, In the case of $n > 1$, Cheng, Hwang and Yang prove that the weak $C^1$ solutions of ( **??**) are class $C^2$, While in the case of $n = 1$, Ritoré provides examples of non regular minimum surfaces. Finally, regarding the low regularity, Bigolin and Serra Cassano demonstrate the lipschitz regularity of the solutions. Instead, the problem of high regularity was completely open. In the works [18] and

[19] (attached to the question) we request as hypothesis the lipschitzianità of the solutions and we prove that they are infinitely differentiable in intrinsic sense.

More precisely, In [18], in the case of $n > 1$, we demonstrate that the lipschitziane viscous solutions of the curvature equation are of class $C^\infty$, while, in [19], in the case of $n = 1$, we prove that they are $C^\infty$ intrinsically. The behavior is therefore radically different in both cases. In fact, if $n > 1$ it is worth a weak condition of Hörmander, the Lie algebra generated by the fields and their first order commutators coincides with the entire space at each point, while if $n = 1$ such condition is violated. The test makes essential use of the possibility of approximating the sub-Riemannian curvature operator with the analogous Riemannian operator, and the approximation results of paragraph (ii) that allow to represent the fundamental solution of an operator sum of squares, as limit of fundamental sotution of riemannian operators.

In [20] an equation of evolution of fronts is studied in a group of steps $s$ that differs from the motion for curvature because it is variational, the so-called total Variation:

$$\partial_t u = \sum_{i=1}^{m} X_i \left( \frac{X_i u}{\sqrt{1 + |\nabla_X u|^2}} \right) \quad \text{in} \quad \mathbb{R}^n \times [0, +\infty[. \tag{4}$$

A result of existence is established for all times of a regular solution of the equation on an bounded convex open set and given to the assigned boundary data. We also prove that the solution tends for $t \to +\infty$ to the solution of the problem of minimal surfaces.

(v) *Minimum of Mumford-Shah's functional.*

The minimum surfaces can also be determined as a set of discontinuities of the minimums of the Mumford-Shah functional in the Riemannian and sub-Riemannian context. In [15] the problem of the existence of minimums is faced with Gamma instruments convergence and is demonstrated a result of convergence of a family of discrete functional to the functional of Mumford-Shah, in a Riemannian space.

Work generalizes to this setting the proof of a conjecture of De Giorgi in the Euclidean case. Next, in [16] the study of the Mumford-Shah functional in the sub-Riemanian metric that shapes the visual cortex.

June 3, 2024

In fede

*Maria Manfredini*